MuMiSTA: An Age-Aware Reservation-Based Random Access Policy

Orhan Tahir Yavascan, Mutlu Ahmetoglu, and Elif Uysal
Communication Networks Research Group (CNG),
Dept. of Electrical and Electronics Engineering, METU, 06800, Ankara, Turkey
{orhan.yavascan,mutlu.ahmetoglu,uelif}@metu.edu.tr

***Abstract*—We introduce Mini Slotted ALOHA and MuMiSTA as novel reservation based random access policies focused on the scalable and timely delivery of status updates in a dense network. In mini slotted ALOHA, each data slot is preceded by multiple mini slots and active users compete for the use of the data slot, by becoming the only user to transmit in one of the mini slots. MuMiSTA is an age-aware modification of mini slotted ALOHA where the set of active users is limited to the users with age larger than a fixed threshold. Users with smaller ages do not congest the channel while conserving energy. In this paper, we derive the throughput of mini slotted ALOHA along with the set of optimal parameters, under finite and infinite node assumptions. We express the ideal number of mini slots in terms of the throughput, depending on the ratio between the lengths of the data slot and the mini slot. A steady state distribution of the number of active nodes under MuMiSTA is derived and an efficient method of obtaining MuMiSTA parameters is described. It is shown that it can approach a ideal round-robin policy with a throughput of 1. In a practical scenario, MuMiSTA is shown to achieve a throughput increase of** 141% **over slotted ALOHA, and an average AoI reduction by** 79%**.**

***Index Terms*—slotted ALOHA, threshold ALOHA, age of information, age threshold, information freshness, random access, age-aware, mini slot, reservation-based.**

I. Introduction

Rapid growth of Internet-of-Things(IoT) systems in recent years has resulted in the development and deployment of numerous real time applications, such as remote sensing and energy scheduling. Age of Information and other information freshness metrics quantify the timeliness of status updates and are more relevant measures of performance in recent real-time applications compared to delay.

In many of the large-scale IoT networks, a centralized multiple access scheme with scheduled access is not practical. Instead, many IoT and MTC applications use random access technologies, like CSMA and slotted ALOHA. Reservation based random access policies have emerged five decades ago in broadcast [1] and satellite [2] communication studies as a compromise between ALOHA and TDMA. It has been studied in the context of ad-hoc networks [3], M2M communications [4], vehicular networks [5] and in satellite networks [6]. Real time applications based on these types of networks would benefit from improved spectral efficiency and information freshness.

In [7] and [8], a slotted ALOHA policy with an age threshold was proposed and rigorously studied. Average AoI has been shown to be reduced by nearly half when an age threshold is used to reduce the number of users that participate in the random access scheme. However, the inherent throughput limitation of slotted ALOHA restricts the achievable amount of AoI improvement even with age-aware threshold and transmission probability modifications as in [7]–[9]. Motivated by this observation, MiSTA [10] has suggested a modified time slot structure in which each data slot is preceded by a mini slot for control and reservation purposes. In [11], a multi-access policy with a framed slot structure is studied in terms of AoI where one of the slots in each frame is used for reservation. In [12], slotted ALOHA and CSMA networks are investigated whereby time horizon is divided into homogeneous smaller mini slots.

In this paper, we introduce a novel random access policy, Multiple Mini Slotted Threshold ALOHA (MuMiSTA). In MuMiSTA, we suggest a reservation based transmission policy by employing multiple mini slots before each data slot, such that the channel is utilized to a greater degree whilst preserving the random access nature.

Our main contributions are the following:

- We describe the novel slot structure of *mini slotted ALOHA* utilizing prefixed mini slots ahead of data slots. Further, we introduce MuMiSTA as an age-aware modification of mini slotted ALOHA.
- We express the throughput of mini slotted ALOHA in the case of a finite network size and a network size that grows to infinity, and derive the maximum achievable throughput along with optimal parameters. We determine the ideal number of mini slots in each time slot such that the throughput of mini slotted ALOHA is maximized.
- We define a truncated state space for a network running MuMiSTA, and express the steady state distribution of the number of active nodes.
- We propose a method with reduced complexity to obtain a set of parameters for MuMiSTA achieving major improvement in terms of throughput and average AoI, compared to slotted ALOHA.

II. System Model and Definitions

We consider n nodes that interact with a common access point (AP) through a random access channel. We adopt the "generate at will" model, where the packets are generated by the nodes immediately before a transmission takes place. The nodes perform transmissions in a synchronized manner. The time horizon is slotted, comprising mini slots and data slots. The ratio between the length of a data slot and a mini slot is denoted as L and is greater than 1. No collision resolution is performed at the destination; if two or more nodes attempt a transmission at the same time, all packets are discarded

This work was supported by the TUBITAK project 22AG019. Orhan T. Yavascan and Mutlu Ahmetoğlu were supported by Turkcell and Turk Telekom, respectively, within the framework of 5G and Beyond Joint Graduate Support Programme coordinated by Information and Communication Technologies Authority.

by the AP. Lost packets are not retransmitted, the nodes continue their random access policies with fresh packets in the next slot. Upon successful transmission, the nodes receive a success feedback (this is necessary for consistency with the generate-at-will model). In the following, we define two reservation based random access policies.

In **Mini Slotted ALOHA** each time slot begins with $W-1$ mini slots, which are used as an opportunity to reserve the remainder of the slot, which we refer to as a data slot (Fig. 1). In the i^{th} mini slot, each active user attempts transmission with probability τ_i, and leaves the active set with probability $1-\tau_i$ for the rest of the time slot. If a user receives an ACK, this means this user was the sole attempter, and the data slot will be reserved for this user. Otherwise, each user (if any) in the set of active users again independently attempts transmission in the next mini slot, with probability τ_{i+1}, and leaves the active set with probability $1-\tau_{i+1}$. If, by the beginning of the data slot, no reservation has yet been made, then each active user attempts to transmit sensor data in the data slot with probability τ_W.

MuMiSTA is an age-aware modification of mini slotted ALOHA that refines the set of active users in order to improve information freshness. If the flow served by a node currently has age below Γ, the node will stay silent. Nodes with ages at or above the age threshold Γ are *active nodes*, and only active nodes are allowed to use the random access mechanism in the form of mini slotted ALOHA described above. Nodes with ages below Γ are passive nodes and stay idle until their AoI reaches this threshold.

Our proposed algorithms fix the number of mini slots in a time slot. Hence, even when a node reserves the data slot in one of the earlier mini slots, it will delay its transmission until the beginning of the data slot. While this slot structure may seem wasteful compared to some other reservation-based slotted ALOHA protocols, it allows passive nodes to sleep for a long time and conserve energy without repeatedly waking up for synchronization purposes.

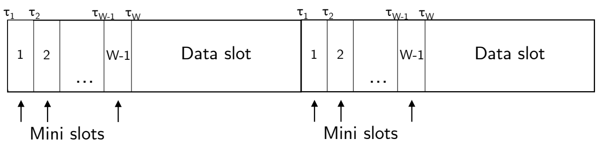

Fig. 1: Diagram of two consecutive time slots, each consisting of multiple mini slots and a data slot.

III. Throughput Analysis of Mini Slotted ALOHA

In this section, we derive a simplified expression for the optimal throughput in a data frame at steady state, according to mini slotted ALOHA. Note that, as in each data slot at most one user can successfully transmit, this is equivalent to the probability of a successful transmission in any given data slot. We denote the expected throughput when there are m active users in the system as $T(m,\{\tau_i\}_{i=1}^W)$, $\{\tau_i\}_{i=1}^W$ is the sequence of transmission probabilities in each mini slot. The net throughput of the system at steady state can be computed by multiplying this value by $\frac{L}{L+W-1}$. This product can be further optimized over the choices of L and W.

A. Throughput of Mini Slotted ALOHA for finite network size

In this section, we compute the expected number of successful transmissions per data slot at steady state in mini-slotted ALOHA.

Theorem 1. *Consider mini-slotted ALOHA (Def. 2) with $W-1$ mini slots, and $m \geq 1$ active users, where the attempt probabilities for active users in mini slots numbered 1 to $W-1$ are, respectively, $\tau_1, \tau_2, \ldots, \tau_{W-1}$, and the attempt probability in the data slot is τ_W. Define $\zeta_j \triangleq m\prod_{i=1}^{j}\tau_i$, for $j=1,\ldots,W$ and $\zeta_{W+1}=0$. The expected number of successful transmissions at the data slot is the following:* [1]

$$T_m = \sum_{j=1}^{W}\left(1-\frac{\zeta_j}{m}\right)^{m-1}(\zeta_j-\zeta_{j+1}) \tag{1}$$

Proof. See Appendix A. □

Throughput of mini slotted ALOHA closely resembles the throughput of slotted ALOHA. We also notice that the use of attempt parameters ζ_j instead of the τ_j sequence results in a clearer narrative. Hence, we use the attempt parameters instead of attempt probabilities in the rest of the paper, unless stated otherwise.

Theorem 2. *Let the sequence $\{D_k(m),\ k,m \in \mathbb{Z}^+\}$ be defined as follows:*

$$D_1(m) = 0, \tag{2}$$

$$D_k(m) = \frac{\left(1-\frac{1}{m}\right)^{m-1}}{\left(1-\frac{D_{k-1}(m)}{m}\right)^{m-1}}, \quad k=2,3,... \tag{3}$$

i) Maximum probability of successful transmission in the data slot of mini slotted ALOHA with $m \geq 2$ active nodes is:

$$T_{max} = D_{W+1}(m). \tag{4}$$

ii) The attempt probabilities that maximize the probability of successful transmission in the data slot are:

$$\tau_j^* = \frac{1-\prod_{i=j+1}^{W+1} D_i(m)^{\frac{1}{m-1}}}{1-\prod_{i=j}^{W+1} D_i(m)^{\frac{1}{m-1}}}, \qquad j=1,2,\ldots,W \tag{5}$$

Proof. See Appendix B. □

As a result of Theorem 2, optimal throughput of mini slotted ALOHA can be computed recursively, for any number of users. For the special case of $W=1$, the policy is equivalent to slotted ALOHA. Maximum throughput of slotted ALOHA is achieved when the expected number of transmissions is equal to 1, corresponding to the throughput $\left(1-\frac{1}{m}\right)^{m-1}$ with $\tau=\frac{1}{m}$, consistent with Theorem 2.

In Fig. 2, the iterative derivation of the $D_k(m)$ sequence is illustrated whereby intersection points on the blue line and the orange line stand for $(D_{k-1}(m), D_k(m))$ and $(D_k(m), D_k(m))$ respectively.

[1] We believe it is obvious to the reader that T_m depends on the transmission probabilities $\{\tau_i\}_{i=1}^W$, so we leave these parameters out of the notation of T_m for simplicity.

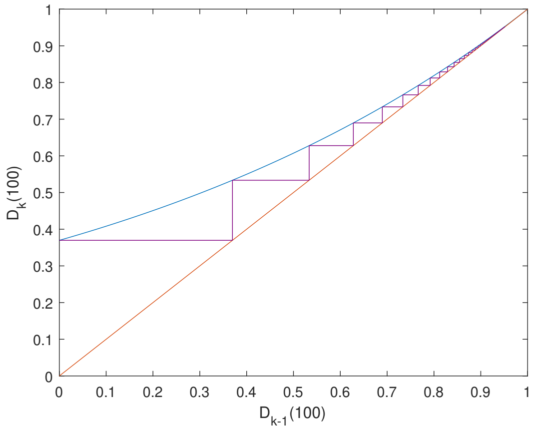

Fig. 2: Iteratively obtaining the maximum throughput of mini-slotted ALOHA for 100 nodes.

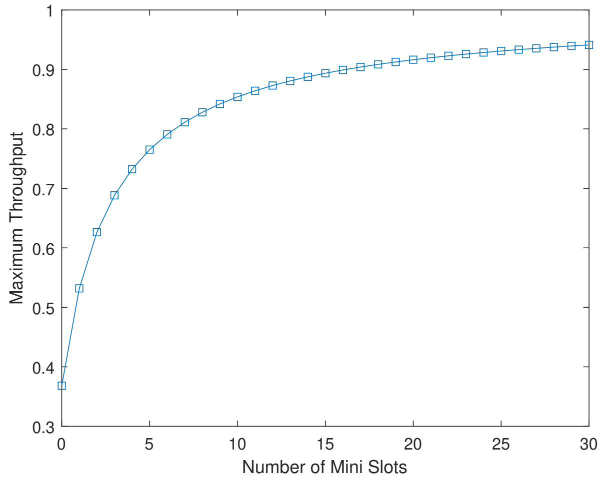

Fig. 3: Maximum throughput of mini slotted ALOHA vs the number of mini slots for 100 users.

B. Infinite User Case

In this section, we consider the asymptotic throughput of mini slotted ALOHA as the network size grows, as commonly done in the analyses of slotted ALOHA systems [13]. We redefine the parameters such that ζ_j's correspond to the expected number of transmissions in the infinite user case; $\zeta_j \triangleq \lim_{m\to\infty} m \prod_{i=1}^{j} \tau_i$. Then, we use (1) to obtain:

$$T_\infty = \lim_{n\to\infty} T_n = \sum_{j=1}^{W} (\zeta_j - \zeta_{j+1})\, e^{-\zeta_j}. \tag{6}$$

Notice that, the special case of slotted ALOHA yields Ge^{-G}, where G is the expected number of transmissions per time slot. In the following, we present the optimal throughput and important characteristics of mini-slotted ALOHA under infinite-node assumption:

Theorem 3. *Let the sequence $\{Q_k,\ k \in \mathbb{Z}^+\}$ be defined as follows:*

$$Q_1 = 0, \tag{7}$$

$$Q_k = \exp(Q_{k-1} - 1), \quad k = 2, 3, \ldots \tag{8}$$

i) Maximum probability of successful transmission in the data slot of mini slotted ALOHA with infinitely many nodes is:

$$T_{max} = Q_{W+1}. \tag{9}$$

ii) The attempt parameters that maximize the probability of successful transmission in the data slot are:

$$\zeta_j^* = \sum_{i=j}^{W} (1 - Q_i), \qquad j = 1, 2, \ldots, W \tag{10}$$

iii) The Q_k sequence satisfies the following limit:

$$\lim_{k\to\infty} k(1 - Q_k) = 2 \tag{11}$$

Proof. See Appendix C. □

Corollary 1. *Maximum throughput of mini-slotted ALOHA in the data slot satisfies the following:*

$$T_{max} = 1 - \frac{2}{W} + o\left(\frac{1}{W}\right), \tag{12}$$

as a result of Theorem 3.

C. Ideal Number of Mini Slots

Increasing the number of mini slots before a data slot increases the probability of a successful reservation of the data slot and therefore, a collision-free transmission in the data slot. However, too many mini slots will reduce the share of time spent on data transmissions and reduce the throughput. The duration of a data slot spans a fraction $\frac{L}{L+W-1}$ of the corresponding time slot. The average throughput of mini slotted ALOHA is derived by multiplying $\frac{L}{L+W-1}$ with T_m. Hence, maximum throughput of mini slotted ALOHA is:

$$T_{max}^{(L,W)} = \frac{L}{L+W-1} D_{W+1}(m) \tag{13}$$

In the following theorem, we derive the optimal number of mini slots in terms of the throughput:

Theorem 4. *Throughput optimal window size W^* is the greatest integer such that $h(W^*) \leq L$ where $h(w)$ is defined as:*

$$h(w) = \frac{D_{w+1}(m)}{D_{w+1}(m) - D_w(m)} - w + 1 \tag{14}$$

Proof. In order to find the optimal value of W in terms of throughput, we study the $T_{max}^{(L,w-1)} \leq T_{max}^{(L,w)}$ statement. This is equivalent to:

$$\frac{LD_w(m)}{L+w-2} \leq \frac{LD_{w+1}(m)}{L+w-1}, \tag{15}$$

which can be rewritten to obtain $h(w) \leq L$. Similarly, $T_{max}^{(L,w+1)} < T_{max}^{(L,w)}$ yields $h(w+1) > L$. □

The h sequence allows us to properly tune W to the size of the data slot and the mini slots. For large L and m values, optimal W can simply be approximated as $\sqrt{2L}$ as a result of Corollary 1. Consequently, the maximum achievable throughput can be approximated to $1 - 2\sqrt{\frac{2}{L}}$.

In the following section, the effect of L on the throughput and average AoI is not repeated throughout the section to simplify the expressions.

IV. Steady State Analysis of MUMISTA

A. Truncated State Space Model

Based on mini slotted ALOHA, MuMiSTA is as follows: At the beginning of each time slot, each node in the network checks whether their AoI has reached the age threshold, Γ. Then, all active nodes perform mini slotted ALOHA in the time slot as described in the previous section. As opposed to pure mini slotted ALOHA, the number of active users is a time-varying stochastic process in MuMiSTA. We define the attempt parameters in terms of n, the number of all nodes: $\zeta_j \triangleq n\prod_{i=1}^{j}\tau_i$, for $j = 1,\ldots,W$ and $\zeta_{W+1} = 0$. Then, the probability of a successful transmission in a time slot with m active nodes is:

$$T_m = \frac{m}{n}\sum_{j=1}^{W}\left(1-\frac{\zeta_j}{n}\right)^{m-1}(\zeta_j-\zeta_{j+1}), \tag{16}$$

from Theorem 1. In this section, we analyze the steady state behaviour of the MuMiSTA policy. We use the *truncated* state space method of [8] in our analysis. The truncated state vector is defined as:

$$\mathbf{A}^\Gamma[t] \triangleq \langle A_1^\Gamma[t] \quad A_2^\Gamma[t] \quad \ldots \quad A_n^\Gamma[t]\rangle \tag{17}$$

where $A_i^\Gamma[t] \in \{1,2,\ldots,\Gamma\}$ is the AoI of source i at time $t \in \mathbb{Z}^+$ truncated at Γ and evolves as:

$$A_i^\Gamma[t] = \begin{cases} 1, & \text{src. } i \text{ updates at time } t-1,\\ \min\{A_i^\Gamma[t-1]+1,\ \Gamma\}, & \text{otherwise.}\end{cases} \tag{18}$$

In the following Theorem, we establish the distribution of m, the number of active users.

Theorem 5. *The truncated MC $\{\mathbf{A}^\Gamma[t], t \geq 1\}$ has the following properties:*

- *i) Given a state vector $\langle s_1 \quad s_2 \quad \ldots \quad s_n\rangle$, its steady state probability depends only on the number of entries that are equal to Γ.*
- *ii) Let P_m be the total steady state probability of states having m active users. Then,*

$$\frac{P_{m+1}}{P_m} = \frac{1-T_m}{T_{m+1}}\frac{n-m}{\Gamma-n+m} \tag{19}$$

Proof. Similar to [8], we establish four classes of types that can precede or succeed a state of the other types. The type sets are:

- $\mathcal{T}_0 \triangleq (M,\{u_1,u_2,\ldots,u_{n-M-1},1\})$
- $\mathcal{T}_1 \triangleq (M+1,\{u_1,u_2,\ldots,u_{n-M-1}\})$
- $\mathcal{T}_2 \triangleq (M+1,\{u_1-1,u_2-1,\ldots,u_{n-M-1}-1\})$
- $\mathcal{T}_3 \triangleq (M,\{\Gamma-1,u_1-1,u_2-1,\ldots,u_{n-M-1}-1\})$

States of types $\mathcal{T}_0$ and $\mathcal{T}_1$ can only be preceded by a state of $\mathcal{T}_2$ or $\mathcal{T}_3$. Further, states of types $\mathcal{T}_2$ and $\mathcal{T}_3$ can only be succeeded by states of types $\mathcal{T}_0$ or $\mathcal{T}_1$. We may use this relationship to write the steady state equations for states of $\mathcal{T}_0$ and $\mathcal{T}_1$:

$$\pi_{\mathcal{T}_1} = \pi_{\mathcal{T}_2}(1-T_{M+1}) + \pi_{\mathcal{T}_3}(M+1)(1-T_M) \tag{20}$$

$$\pi_{\mathcal{T}_0} = \pi_{\mathcal{T}_2}\frac{T_{M+1}}{M+1} + \pi_{\mathcal{T}_3}T_M \tag{21}$$

In the following, we test the equiprobability property stated in Theorem 5(i), by assigning π_m to be the steady state probability of a state with m active users. Replacing $\pi_{\mathcal{T}_1}$ and $\pi_{\mathcal{T}_2}$ with π_{M+1} and $\pi_{\mathcal{T}_0}$ and $\pi_{\mathcal{T}_3}$ with π_M yields:

$$\pi_{M+1} = \pi_{M+1}(1-T_{M+1}) + \pi_M M(1-T_M), \tag{22}$$

$$\pi_M = \pi_{M+1}\frac{T_{M+1}}{M+1} + \pi_M T_M. \tag{23}$$

Equations (22) and (23) are reduced to the same equation below:

$$\frac{\pi_{m+1}}{\pi_m} = (m+1)\frac{1-T_m}{T_{m+1}}. \tag{24}$$

Further, please note that the state where all sources are truncated is accessible by all states and is aperiodically recurrent; showing the uniqueness of the steady state solution. As a result of this, (24) is sufficient to describe the steady state solution. The total number of states corresponding to π_m is the number of recurrent states with m sources at truncated age Γ:

$$N_m = \binom{n}{m}\frac{(\Gamma-1)!}{(\Gamma-n-1+m)!} \tag{25}$$

Steady state probability of having m active sources can be derived by multiplying N_m with π_m:

$$P_m = N_m\pi_m \tag{26}$$

provides the steady state solution. □

Corollary 2. *The expected throughput of a MuMiSTA network at steady state can be computed by Theorem 5:*

$$T = E[T_M] = \sum_{m=1}^{n} P_m T_m \tag{27}$$

PMF of the active nodes can also be used to derive the expected average AoI of the nodes at the steady state. The age of a passive node is uniformly distributed in the set $\{1,2,\ldots,\Gamma-1\}$ as a result of the first property and therefore, the expected age of a passive node is $\frac{\Gamma}{2}$. The expected age of an active node depends on the expected time until the next successful transmission by that node upon becoming active. We note that, while the steady state probability of truncated states was shown to be independent of the age of the passive nodes, the expected time until a successful transmission depends on the age of passive nodes and requires a solution of high complexity $O(\Gamma^n)$, shown in [7]. However, the probability of successful transmission by any given active node while there are m symmetric active nodes is, by symmetry, $\frac{T_m}{m}$. Considering the fact that the number of active nodes changes very slowly and that the number of active nodes will be probabilistically clustered around the local maxima of the PMF of the active nodes, we approximate the expected age of an active node while there are m active nodes to $\Gamma + \frac{m}{T_m}$. Consequently, we obtain the following approximation on the average AoI:

Corollary 3. *The expected AoI of a MuMiSTA network at the steady state can be approximated as:*

$$\Delta \simeq \frac{\Gamma}{2} + \frac{\Gamma}{2}E\left[\frac{M}{n}\right] + E\left[\frac{M^2}{nT_M}\right] \tag{28}$$

The approximation is justified by the convergence of the ratio of the active nodes as $n \to \infty$ [8], and the accuracy of the approximation is illustrated in Sec. V.

B. Design of MuMiSTA Parameters

Theorem 5 can be used to derive the local maxima of the distribution of the number of active nodes. Using (19), we define:

$$\gamma(m) \triangleq (n-m)\frac{1-T_m+T_{m+1}}{T_{m+1}}, \tag{29}$$

such that $P_{m+1} > P_m$ if and only if $\gamma(m) > \Gamma$. Let $m_0 \triangleq \gamma^{-1}(\Gamma)$, then either $\lceil m_0 \rceil$ or $\lfloor m_0 \rfloor$ shall be a local extrema. In order to find a good set of parameters with low complexity, we may search through suitable m_0 values. For each m_0, we select attempt parameters that maximize T_m according to Theorem 2. The age threshold Γ is chosen as $\lceil \gamma(m_0) \rceil$. Finally, the average AoI can be approximated using Corollary 3. Selection of m_0 with lowest AoI approximation results in an acceptable set of parameters for MuMiSTA with O(n) complexity.

V. Numerical Results and Discussion

In this section, we present our results for MuMiSTA policies with varying number of mini slots, of 200 nodes. The ratio between the length of data slots and mini slots are assumed to be $L = 500$, with lengths of a mini slot and a data slot being set to $10\mu s$ and $5ms$, respectively.

W(Window Size)	Thrghpt.	AAoI (ms)	$E[m]$	Γ
Slotted ALOHA	0.3678	2720.5	200	–
1 (TA [8])	0.3637	1491.9	54.88	399
2 (MiSTA [10])	0.5255	1008.5	45.18	295
8	0.7969	641.2	24.66	218
32	0.8882	567.8	11.34	201
Round-robin	1	502.5	–	–

TABLE I: A comparison of round-robin, slotted ALOHA and MuMiSTA policies with different window sizes with respect to throughput, average AoI, expected number of active nodes at steady state and age threshold, 200 nodes, $t_d = 5ms$, $t_m = 10\mu s$.

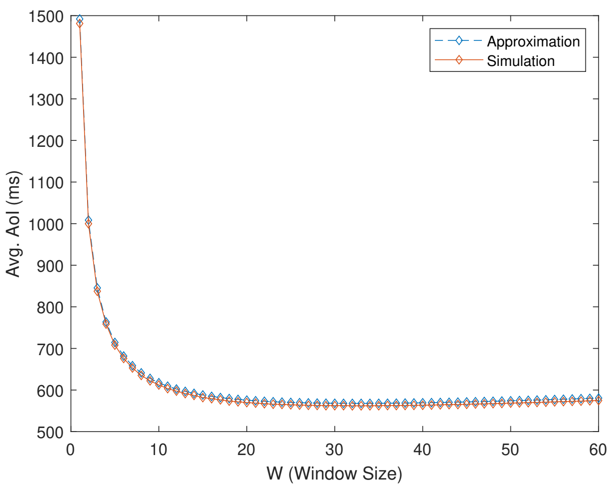

Fig. 4: Average AoI of mini slotted ALOHA vs W for 200 users, $t_d = 5ms$, $t_m = 10\mu s$.

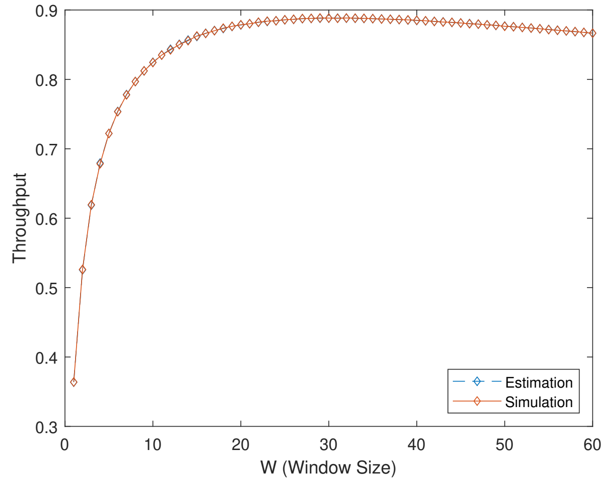

Fig. 5: Throughput of mini slotted ALOHA vs W for 200 users, $t_d = 5ms$, $t_m = 10\mu s$.

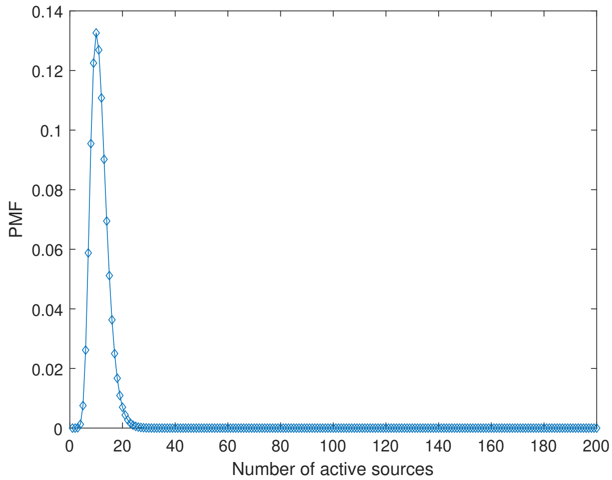

Fig. 6: PMF of the number of active users, $n = 200$, $W = 32$ slots.

In Table I, the performance of MuMiSTA policies are compared to a slotted ALOHA policy and an ideal round-robin policy with no overhead. In Fig. 5, simulation results are compared to the estimation expression derived in Corollary 2 and it is shown that the throughput increases sharply for smaller values of W. As $h(W)$ exceeds L (see Theorem 4), the length of mini slots hinders the effectiveness of MuMiSTA and decreases the throughput. In Fig. 4, the average AoI of MuMiSTA is illustrated against the number of mini slots and the approximation presented in Corollary 3 is shown to be accurate within 1% of simulation results. Both throughput and the average AoI are optimized at 32 mini slots, due to the value of $L = 500$. If MuMiSTA with 32 mini slots is adopted according to Sec. IV-B, expected ratio of active users is less than 6% (Fig. 6), while the probability of more than 15% of the nodes being active at the steady state is less than $.01\%$. The throughput is increased by 141% and average AoI is decreased by 79% in comparison to slotted ALOHA policy. Note that even an ideal round-robin policy decreases the average AoI by at most 81%, compared to slotted ALOHA [14].

Overall, MuMiSTA grants great improvements to the performance of a homogeneous random access setting in terms of information freshness and energy consumption by increasing the throughput and reducing the number of active users. These improvements, however, come at the cost of a more complicated reservation based slot structure compared to slotted ALOHA. Consequently, depending on the system requirements, MuMiSTA has the potential to become a favorable alternative to the traditional random access schemes in the future of IoT applications. Future work may include the analysis of MuMiSTA for infinite nodes, investigation under lossy channels or utilization with SIC techniques.

Appendix A
Proof of Theorem 1

We use proof by induction. For $W = 1$, the system is equivalent to slotted ALOHA and the throughput is $T(n, \tau_1) = n\tau_1 \left(1-\tau_1\right)^{n-1}$; a successful transmission occurs iff there is exactly one user making a transmission attempt.

Next, assume $W \geq 2$. Let there be k nodes attempting transmission in the first mini slot, with probability $\binom{m}{k}\tau_1^k(1-\tau_1)^(m-k)$. A successful reservation happens if $k = 1$ and no successful transmission happens in the data slot if $k = 0$. If $k \geq 2$, the conditional throughput of the data slot is:

$$T(k, \{\tau_i\}_{i=2}^{W}) = \sum_{j=2}^{W} \left(1 - \frac{\zeta_j}{m\tau_1}\right)^{k-1} \left(\zeta_j - \zeta_{j+1}\right) \frac{k}{m\tau_1}, \tag{30}$$

from the induction claim. Then, the throughput of the data slot can be found as:

$$T_m = \zeta_1(1-\frac{\zeta_1}{m})^{m-1} + \sum_{k=2}^{m} \binom{m}{k} \tau_1^k (1-\tau_1)^{m-k} T(k, \{\tau_i\}_{i=2}^{W}) \tag{31}$$

Finally, (1) can be obtained from (30) and (31) to confirm the induction hypothesis.

Appendix B
Proof of Theorem 2

To maximize (1), we evaluate $\frac{\partial T_m}{\partial \zeta_j} = 0$, which is equivalent to:

$$\left(\frac{A_{j-1}}{A_j}\right)^{m-1} = m - (m-1)\frac{A_{j+1}}{A_j}, \tag{32}$$

where $A_j \triangleq 1 - \frac{\zeta_j}{m}$. We derive that $\left(\frac{A_{j-1}}{A_j}\right)^{m-1} = D_j(m)$ and $A_j = \prod_{i=j}^{W+1} D_i(m)^{\frac{1}{m-1}}$ to prove the second part. First part follows from induction on W using the optimal parameters in second part.

Appendix C
Proof of Theorem 3

First two parts follow from Theorem 2 as m tends to infinity. To prove the third part, we define R_k sequence as follows:

$$R_k = \frac{1}{1-Q_k} - \frac{1}{1-Q_{k-1}} = \frac{1}{1-\exp(Q_{k-1}-1)} - \frac{1}{1-Q_{k-1}}. \tag{33}$$

Then, R_k can be expressed as a function of Q_{k-1}:

$$R_k = g(1 - Q_{k-1}), \tag{34}$$

where g function is defined as:

$$g(x) = \frac{1}{1-\exp(-x)} - \frac{1}{x} = \frac{x - 1 + \exp(-x)}{x - x\exp(-x)}. \tag{35}$$

Further, the limit of g as x goes to 0 (or as Q_{k-1} goes to 1) is $1/2$, found from the L'Hôpital's rule. Then,

$$\lim_{k\to\infty} \frac{1}{1-Q_k} - \frac{1}{1-Q_{k-1}} = \lim_{k\to\infty} R_k = \frac{1}{2}. \tag{36}$$

Finally, Stolz–Cesàro theorem can be used to derive the following limit:

$$\lim_{k\to\infty} \frac{1}{k(1-Q_k)} = 2, \tag{37}$$

from which the third part follows.

References

[1] W. Crowther, R. Rettberg, D. Walden, S. Ornstein, and F. Heart, "A system for broadcast communication: Reservation-aloha," in *Proc. 6th Hawaii Int. Conf. Syst. Sci*, pp. 596–603, 1973.

[2] L. G. Roberts, "Dynamic allocation of satellite capacity through packet reservation," in *Proceedings of the June 4-8, 1973, national computer conference and exposition*, pp. 711–716, 1973.

[3] S. Asadallahi and H. H. Refai, "Modified r-aloha: Broadcast mac protocol for vehicular ad hoc networks," in *2012 8th International Wireless Communications and Mobile Computing Conference (IWCMC)*, pp. 734–738, IEEE, 2012.

[4] F. Vázquez-Gallego, J. Alonso-Zárate, and L. Alonso, "Reservation dynamic frame slotted-aloha for wireless m2m networks with energy harvesting," in *2015 IEEE International Conference on Communications (ICC)*, pp. 5985–5991, IEEE, 2015.

[5] F. Borgonovo, A. Capone, M. Cesana, and L. Fratta, "Rr-aloha, a reliable r-aloha broadcast channel for ad-hoc inter-vehicle communication networks," in *Proceedings of Med-Hoc-Net*, vol. 2002, 2002.

[6] M. Lee, J.-K. Lee, J.-J. Lee, and J. Lim, "R-crdsa: Reservation-contention resolution diversity slotted aloha for satellite networks," *IEEE communications letters*, vol. 16, no. 10, pp. 1576–1579, 2012.

[7] D. C. Atabay, E. Uysal, and O. Kaya, "Improving age of information in random access channels," in *Proc. AoI Workshop in conj. with IEEE INFOCOM*, July 2020.

[8] O. T. Yavascan and E. Uysal, "Analysis of slotted aloha with an age threshold," *IEEE Journal on Selected Areas in Communications*, vol. 39, no. 5, pp. 1456–1470, 2021.

[9] X. Chen, K. Gatsis, H. Hassani, and S. S. Bidokhti, "Age of information in random access channels," *arXiv:1912.01473*, 2019.

[10] M. Ahmetoglu, O. T. Yavascan, and E. Uysal, "Mista: An age-optimized slotted aloha protocol," *IEEE Internet of Things Journal*, vol. 9, no. 17, pp. 15484–15496, 2022.

[11] Q. Wang and H. Chen, "Age of information in reservation multi-access networks with stochastic arrivals," *IEEE International Symposium on Information Theory (ISIT)*, 2022.

[12] I. Kadota and E. Modiano, "Age of information in random access networks with stochastic arrivals," in *IEEE INFOCOM 2021-IEEE Conference on Computer Communications*, pp. 1–10, IEEE, 2021.

[13] V. Anantharam, "The stability region of the finite-user slotted aloha protocol," *IEEE Transactions on information Theory*, vol. 37, no. 3, pp. 535–540, 1991.

[14] R. D. Yates and S. K. Kaul, "The age of information: Real-time status updating by multiple sources," *IEEE Transactions on Information Theory*, vol. 65, no. 3, pp. 1807–1827, 2018.